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Discrete-Valued Inversion of Gravity Data over the Voisey's Bay Ovoid Using Fuzzy C-means Clustering

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ABSTRACT

Discrete values can be imposed within a physical property inversion to improve the distinct geologic boundaries in the recovered models. While discrete values can be, and have been, imposed in many ways, we apply the guided fuzzy c-means (FCM) clustering technique to a Tikhonov regularized inversion for use with either gravity or gravity gradient data. By using guided FCM, we are able to maintain spatial flatness for a cohesive model, while the density contrast is clustered around values dictated by prior geologic information. In this paper, we discuss the details of the algorithm, as well as its use with binary problems, where the density contrast is restricted to either a background or anomalous value. Additionally, we illustrate the methodology using the gravity data over the Ovoid deposit at Voisey's Bay in Labrador, Canada.

INTRODUCTION

Inversions are commonly used in geophysics to produce models of the subsurface from geophysical data, but these models are rarely an accurate reflection of the geology. While l2-norm-based objective functions recover smooth physical property models, l1- or lp-norms can recover unrealistically blocky models. In order to recover realistic, yet distinct units within our models, we investigate the use of discrete physical property values within the inversion.

Several methods for imposing discrete values on inverse problems for both gravity and gravity gradiometry exist. Camacho et al. (2000) use the method of "growing bodies" for gravity data, which starts with an initial guess of the anomalous body and a known density contrast, then allows for the body to grow into a suitable solution. Similarly, Uieda and Barbosa (2011, 2012) use a "seeds" method for gravity gradient data, where they start by planting seeds with provided density contrast values at different locations within the model, then allow the anomalous bodies to expand around the seeds. Using two separate approaches, both applied to gravity data, Krahenbuhl and Li (2006, 2009) first implement a genetic algorithm, then combine the genetic algorithm with quenched simulated annealing for a hybrid method. In a similar approach, but for either the individual or joint inversion of gravity and gravity gradient data, Capriotti et al. (2015) apply quenched simulated annealing. While each of these previous methods have been successful in their own way, they all share limitations in model size and number of data, due to their computational complexity and memory requirements associated with computing responses for individual cells within the model.

To overcome these limitations we apply the guided fuzzy c-means clustering (FCM) technique (Sun and Li, 2015) to the discrete-valued, Tikhonov regularized inversion of gravity (Maag and Li, 2015) or gravity gradiometry data (Maag and Li, 2016). This method, applied specifically for binary problems, guides the density contrast values to cluster into one of two distinct regions, the background or anomalous density bodies. By using FCM, we are able to approximate the discrete values by a continuous variable and overcome the previous model-size and number of data limitations by using derivative-based minimization techniques such as wavelet compression presented in Li and Oldenburg (2003).

The discrete-valued, guided FCM inversion methodology has proven successful for synthetic examples for both surface and borehole gravity data, as shown in Maag and Li (2015), and gravity gradiometry data applied to the imaging of salt in Maag and Li (2016). Here, we demonstrate the methodology with a field data set, specifically, the gravity data collected over the Ovoid deposit at Voisey's Bay, in Labrador, Canada. We will first provide a brief overview of the methodology, and its use in situations with a heterogeneous anomalous density contrast. Then, we examine the results of inverting the gravity data over the Ovoid body at Voisey's Bay both with and without the clustering, demonstrating the improvements that can be made on the recovered model.

METHODOLOGY

Incorporating the guided fuzzy c-means clustering into a Tikhonov regularized inversion simply requires the addition of the FCM objective function (ϕ_{FCM}) to the classic objective function,

$$\phi = \phi_d + \beta\phi_m + \lambda\phi_{FCM} \tag{1}$$

where ϕ_d is the classic measure of data misfit, β the Tikhonov regularization parameter, ϕ_m the model objective function, and λ a parameter that weights the influence of the clustering against the other terms in the objective function. When equation 1 is minimized, we obtain the recovered subsurface model,

$$\boldsymbol{m} = \boldsymbol{\Delta\rho} \cdot \boldsymbol{\tau} \tag{2}$$

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which can be defined as the product of a user defined density contrast model, $\boldsymbol{\Delta\rho}$, and the discrete model, $\boldsymbol{\tau}$. This discrete model consists of only zeros and ones, that identify the background and anomalous density contrast regions, respectively. Using this definition for $\boldsymbol{m}$ allows the density contrast to be either a constant value, or vary spatially. We call this a reduction to binary, because it allows even complex density contrast models to be solved as a simple binary, or two cluster problem.

Because the density contrast, $\boldsymbol{\Delta\rho}$, is a scalar multiplier at each location, we are able to solve equation 1 directly for $\boldsymbol{\tau}$, rather than $\boldsymbol{m}$. To do this we must adjust some of the components of the total objective function (equation 1). We are able to maintain the classic measure of data misfit,

$$\phi_d = \| \boldsymbol{W}_d(\boldsymbol{Gm} - \boldsymbol{d}) \|_2^2 \quad (3)$$

where $\boldsymbol{W}_d$ is the data weighting matrix, $\boldsymbol{G}$ the gravity or gravity gradiometry sensitivity matrix, and $\boldsymbol{d}$ the observed data. The model objective function,

$$\phi_m = \| \boldsymbol{W}_m \boldsymbol{S}(\boldsymbol{\tau} - \boldsymbol{\tau}_{ref}) \|_2^2 \quad (4)$$

does however requires some adjustments to be computed directly on the discrete model, $\boldsymbol{\tau}$. With the change of $\boldsymbol{m}$ to $\boldsymbol{\tau}$, we also must replace the reference model $\boldsymbol{m}_{ref}$ with $\boldsymbol{\tau}_{ref}$, but the other components, $\boldsymbol{W}_m$, the model weighting matrix, and $\boldsymbol{S}$, the depth weighting term, remain the same. By computing directly on the discrete model, we are able to maintain spatial smoothing, inherent in an l_2-norm based inversion, to create a cohesive model with distinct units, determined by the clustering. The FCM objective function,

$$\phi_{FCM} = \sum_{j=1}^{M}\sum_{k=1}^{C} u_{jk}^{q} \| \tau_j - v_k \|_2^2 + \eta \sum_{k=1}^{C} \| v_k - t_k \|_2^2 \quad (5)$$

which is the sum of two terms, is also altered to be computed directly on $\boldsymbol{\tau}$. The first term begins with a sum over all the model parameters, M, then a sum over the clusters, C, followed by the membership function u_{jk}, which contains the likelihood that each model parameter belongs to each cluster, raised to a fuzziness factor, q. Because we are using fuzzy clustering, each model parameter is allowed to belong to both clusters. The membership function is multiplied by the squared l_2-norm of the current discrete model, τ_j, and cluster centre, v_k, which are the values determined by the clustering, to have a model cluster around. We want the cluster centres to be 0 for the background and 1 for the anomalous density regions. To push the cluster centres, v_k, towards our known cluster centres, t_k, we use the guiding term, which contains the squared l_2-norm of these two values, and is weighted against the rest of the objective function by η.

In order to solve equation 1, for a discrete model, $\boldsymbol{\tau}$, we use an iterative process that first updates the membership function, u_{jk}, then the cluster centres, v_k, and finally the discrete model. The process is repeated until the algorithm reaches the convergence criteria. It should be noted that, while $\boldsymbol{\tau}$ ideally only contains values of 0 and 1, throughout the iterative process, because we are using fuzzy clustering, the discrete values are approximated as a continuous variable and may range between 0 and 1. By doing this we are able to take advantage of derivative-based minimization techniques that allow us to solve larger problems more efficiently. This is one of the major advantages of the algorithm.

VOISEY'S BAY

To understand the behavior of the algorithm, we apply the methodology to the ground gravity data set collected over the Ovoid deposit at Voisey's Bay, located on the northeast coast of Labrador, Canada. The original purpose of this survey was to recover regional information about the entire Nickel-Copper-Cobalt deposit, but the Ovoid massive sulphide deposit is easy to distinguish. Both Oldenburg et al. (1998) and Ash et al. (2006) were able to recover spatially consistent and well defined models of the Ovoid deposit, but the models suffer from spatial smoothing, and the inability to recover the expected density contrast value. We attempt to overcome these issues using the guided FCM inversion.

The gravity data have previously undergone a complete Bouguer correction. We use an error level of 3% plus 0.01 mGal, which is estimated in Oldenburg et al. (1998), along with a constant density contrast, $\boldsymbol{\Delta\rho}$, for the Ovoid deposit of 1.7 g/cc, given that the maximum density value of the deposit is 4.5 g/cc and the local background is 2.8 g/cc. Despite knowing the Ovoid deposit is unlikely to have a uniform density contrast, we do not have enough information to estimate the density contrast distribution within the deposit, and as such, can expect some amount of uncertainty in our recovered model.

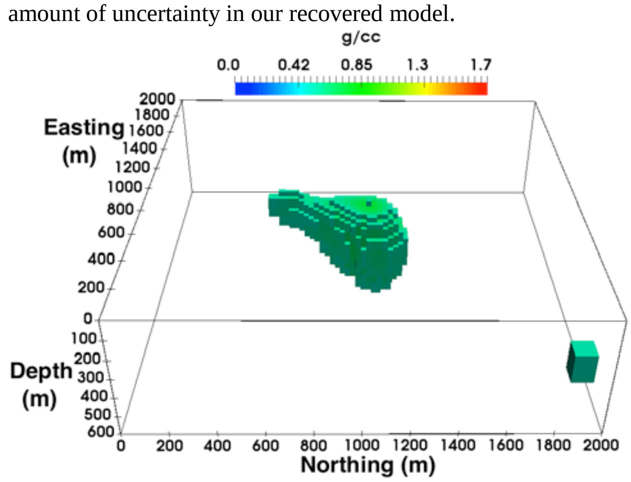


Figure 1: Recovered model from the inversion without clustering of the Voisey's Bay gravity data over the Ovoid deposit.

To begin, we first invert the data without the clustering to recover the models shown in Figures 1 and 2. Figure 1 show a 3-D volume of the recovered model with a cut off of everything below 0.5 g/cc. We see from this view that the deposit seems to have a tail extending to the west. We can also look at the three cross sections through the model shown in Figure 2. While each of these show the general position of the deposit, the models are spatially smooth, and have only recovered a maximum density contrast value of around 1.4 g/cc. These results are consistent with those from previous inversions of the data.

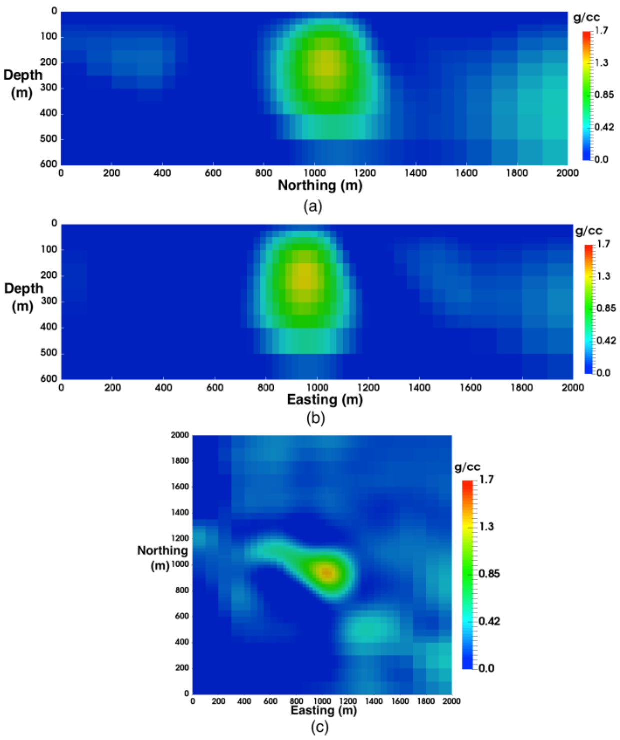


Figure 2: Cross sections through the recovered model from the inversion without clustering of the Voisey's Bay gravity data over the Ovoid deposit, with (a) located at 900 m Easting, (b) located at 987.5 m Northing, and (c) located at 200 m depth.

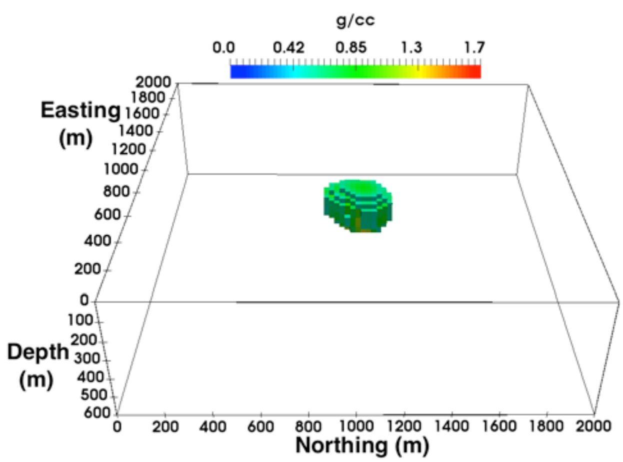


Figure 3: Recovered model from the inversion with clustering of the Voisey's Bay gravity data over the Ovoid deposit.

To improve our recovered model, we invert the data using the guided FCM methodology and obtain the recovered model displayed in Figures 3 and 4. If we first compare the volumes in Figure 1 and 3, we see that the recovered Ovoid region using clustering is more compact and no longer has the tail extending to the west. Additionally, we can compare the cross sections through the recovered models and see that, with clustering, the Ovoid body is more compact, and the recovered density contrast values have reached the known value of 1.7 g/cc in the centre of the deposit. We also see major improvement in the recovery of the bottom of the deposit, which has a known depth of around 250 m. We have not eliminated all the spatial smoothness from the recovered model, and there is still a ring of smoothed values surrounding the recovered Ovoid model. This is due to our use of a uniform density contrast.

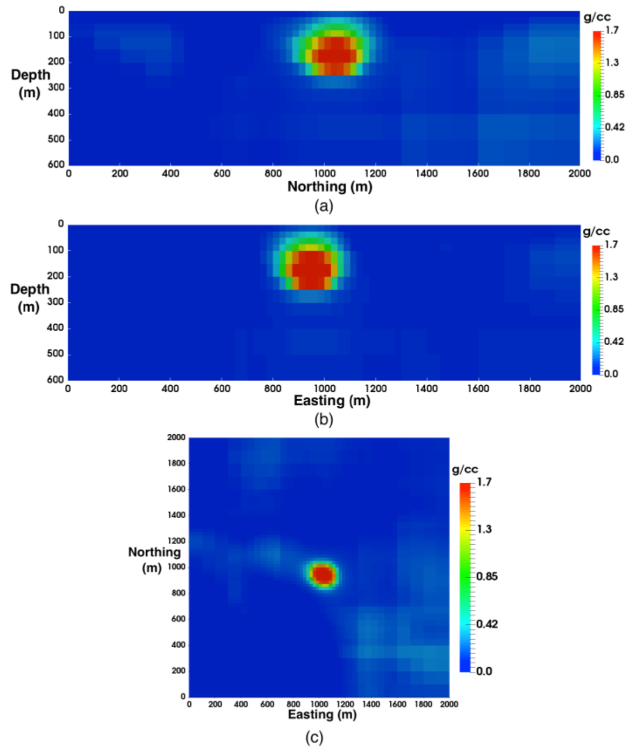


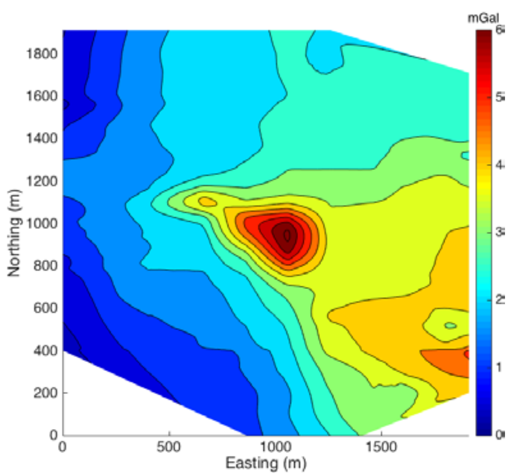
Figure 4: Cross sections through the recovered model from the inversion with clustering of the Voisey's Bay gravity data over the Ovoid deposit, with (a) located at 900 m Easting, (b) located at 987.5 m Northing, and (c) located at 200 m depth.

While the clustering inversion has been able to recover better density contrast values, and a more compact structure for the Ovoid deposit, it is good to compare the observed and predicted data (Figure 5) to make sure the inversion has not forced regions into the wrong cluster. We can observe a strong similarity between the observed and predicted data, and should make note that because we are estimating our discrete values as continuous, we are able to maintain the regional trend within our data.

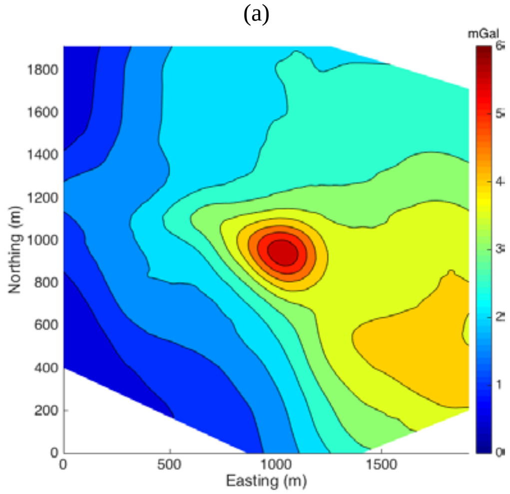
CONCLUSIONS

We have presented the application of the guided fuzzy c-means clustering technique to the discrete-valued inversion of gravity or gravity gradient data. The algorithm overcomes the previous limitations of other discrete-valued inversions by using the fuzzy clustering to approximate our discrete values as a continuous variable, allowing us to implement derivative-based minimization techniques. Additionally, we can solve for both simple binary models, or more complicated density contrast distributions by reducing our problem to binary and solving for a discrete model.

To test the algorithm, we applied it to the gravity data set collected over the Ovoid massive sulphide deposit at Voisey's Bay. The recovered model from the clustering inversion was compared to one from an inversion without clustering. The clustering was able to produce a compact structure, with minimal spatial smoothing due to an estimation of a constant density contrast value, as well as recover the appropriate density contrast value for the deposit. Overall, the addition of the clustering to the inversion of discrete values is able to efficiently recover appropriate physical property values, as well as a spatially distinct, but cohesive model.



(a)



(b)

Figure 5: Comparison of the (a) observed data and (b) predicted data from the inversion without clustering.

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