A Matlab program to invert 1D Spectral Induced Polarization data for Cole-Cole model including electromagnetic effects

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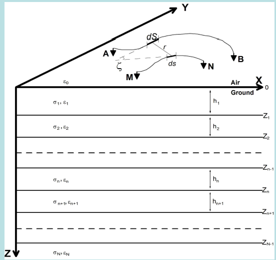
Introduction

At low frequencies, electromagnetic (EM) coupling between the transmitter, the receiver and the ground and normal polarization effects of the subsurface material have similar functional behaviour with respect to the conductivity of the earth and their combined effects are recorded in an Spectral Induced Polarization (SIP) survey. EM coupling is a major impediment in the interpretation of induced polarization (IP) data. A forward modeling code which calculates the mutual impedance in different frequencies of 1D ground layers for Cole-Cole model and different grounded electrode arrays is developed by Ingeman-Nielsen and Baumgartner (2006). We developed an 1D inversion of IP and EM coupling integral according to forward modeling code. A homotopy method is applied to overcome the local convergence of Gauss-Newton and Quasi Newton methods.

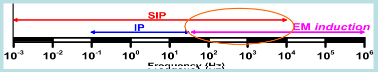
Mutual impedance

According to Sunde (1968), the EM coupling , Z(ω), between two grounded wires in an arbitrary configuration on the surface of the earth can be calculated as an integration of mutual impedances of virtual dipoles along the paths of the wires:

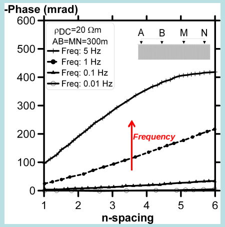


$$Z(\omega)=\int_{A}^{B}\int_{M}^{N}\left[P(r)\cos\xi+\frac{\partial^2 Q(r)}{\partial S\partial s}\right]dsdS$$

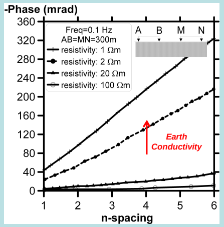
Q and P are often referred to as the grounding function and coupling function, respectively. These are depend on the generalized reflection coefficients and the electromagnetic properties of each subsurface layer.



Diope-Dipole array *(Partha & Oldenburg, 2001):*

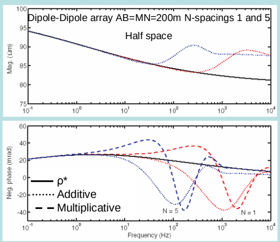


EM coupling increases with the dipole length and dipole separation and with conductivity and frequency. For deep exploration, the dipoles and their separation must be large; hence, the operational frequency is usually low to avoid EM coupling.



Removal of EM coupling

- Measuring SIP data at frequencies low enough that any EM coupling is either negligible or predictable. However, elimination of high frequency IP spectrum, eliminates the important information.
- Fitting a linear or quadratic equation to the measurements at two or three low frequencies. However, IP phase is not always constant, nor linear, and can vary depending on various factors such as earth layer texture.
- Fit the phase spectrum with a multiplication or addition of two Cole–Cole dispersions. However, two figures show that for larger N-spacings, the applicability of the method is reduced accordingly.
- Partha and Oldenburg (2001) suggested the observed response could be approximately described by the infinite-frequency-conductivity (σ_∞)-based EM-coupling multiplied by a complex and frequency-dependent function describing the complex resistivity dispersion (hence the multiplicative approach).



Ingeman-Nielsen and Baumgartner (2006)

Inversion procedure

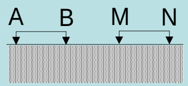
A homotopy method is designed to overcome the local convergence of normal iterative methods.

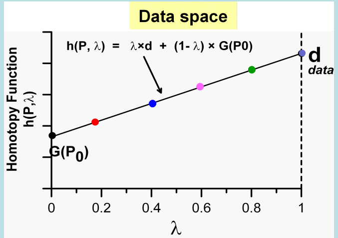
Homotopy theory

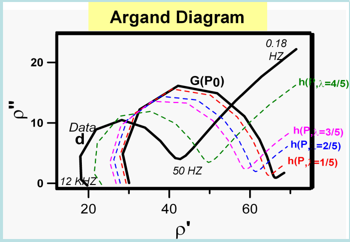
Synthetic Example: Half space

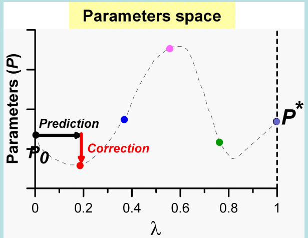
Dipole-Dipole: AB=MN=50m, *N=1*

Freq. range: [0.183 - 12000] Hz









Homotopy algorithm

Gauss-Newton and Quasi-Newton methods allow using for minimization of the misfit function in each step of homotopy inversion:

$$\mathbf{p}_{k+1}^{j}=\mathbf{p}_{k}^{j}-\varepsilon_k\left(\mathbf{G}_k^T\mathbf{C}_{\mathbf{d}_0\mathbf{d}_0}^{-1}\mathbf{G}_k+\mathbf{C}_{\mathbf{p}_0\mathbf{p}_0}^{-1}\right)^{-1}\times\left\{\mathbf{G}_k^T\mathbf{C}_{\mathbf{d}_0\mathbf{d}_0}^{-1}\left[\mathbf{g}(\mathbf{p}_k)-\left(\left(1-\frac{j}{N}\right)\mathbf{g}_0(\mathbf{p}_0)+\frac{j}{N}\mathbf{d}_{obs}\right)\right]+\mathbf{C}_{\mathbf{p}_0\mathbf{p}_0}^{-1}\left(\mathbf{p}_k-\mathbf{p}_{prior}\right)\right\}$$

$\mathbf{p}_{prior}$: Initial estimation for the parameter vector p=$\left[\hat{\rho}_{0i},\hat{m}_i,\hat{c}_i,\hat{\tau}_i,\hat{h}_i\right]^T$

($\hat{\rho}=\log(\rho)$, $\hat{m}=\log(\frac{m}{1-m})$, $\hat{c}=\log(\frac{c}{1-c})$, $\hat{\tau}=\log(\tau)$ and $\hat{h}_i=\log(h_i)$)

$\mathbf{d}_{obs}$: Input data consist of amplitude and phase at a number of frequencies for different spacings of array sounding

C_{p0p0}: The parameters covariance matrix

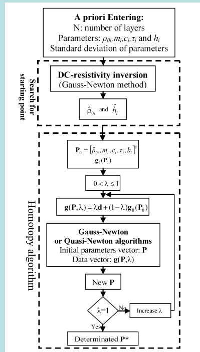
C_{p0p0}: The data covariance matrix

G_k :The derivative matrix of the data in iteration step k

g(p): forward model

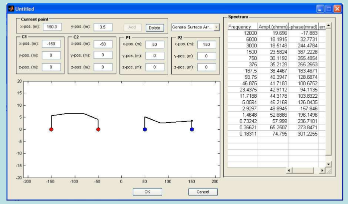
ε_k : is an ad-hoc (real, positive) value adjusted to force the algorithm to converge rapidly

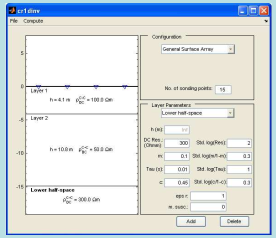
$\lambda_j=\frac{j}{N}$ where λ is the homotopy parameter



Inversion program facilities

Data entering





Prior parameters entering

Results of synthetic data

17 frequencies in the [0.183 12 000] Hz range with a logarithmic step of SIP FUCHS-II equipment (Radic Research).

Example 1:

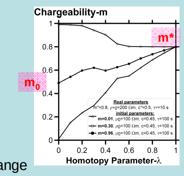
- Dipole-Dipole array, AB=MN=50 m and *n=1 to 5.*

-Transmitter and receiver cables are in same line.

-The number of parts in λ-direction is 10

	layers	Parameters				Standard deviation of parameters				RMS error (%)
		ρ_0^*(Ωm)	*m**	*c**	*τ*(s)*	ρ_0^*	*m**	*c**	*τ*(s)*	
Real values	1	50	0.8	0.3	0.1	1	0.5	0.5	1	0.3
Inversed values		49.98	0.799	0.30	0.099					

Different trajectories in the space of (λ, m) are converged into single response.



Example 2:

- Schlumberger array, the transmitter lines AB change from 0.6 to 60 meter.
- Transmitter and receiver cables are in same line.
- The homotopy inversion consist 5 steps.
- The Quasi-Newton is considered for misfit minimization.

	layers	Parameters					Standard deviation of parameters					RMS error (%)
		ρ_0^*(Ωm)	*m**	*c**	*τ*(s)*	*h*(m)*	ρ_0^*	*m**	*c**	*τ**	*h**	
Real values	1	50	0.3	0.3	1	5	1	0.5	0.5	1	0.1	2.6
Inversed values		49.9	0.30	0.30	0.97	4.96						
Real values	2	300	0.05	0.8	0.01	-	1	0.5	0.5	1	-	
Inversed values		299.7	0.05	0.77	0.01	-						